TESTING SCHRODINGER'S PARADOX WITH A MICHELSON INTERFEROMETER

Evan Harris WALKER

U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, USA

E.C. MAY, S.J.P. SPOTTISWOODE and T. PIANTANIDA

SRI International, Menlo Park, California, USA

The Schrödinger paradox points out that quantum mechanics predicts a linear superposition of states even for macroscopic objects prior to measurement. However, at the macroscopic level of ordinary objects it has not been possible to maintain the phase correlations needed to demonstrate or disprove the reality of such a superposition of states as opposed to the mixture of states. Without such a quantum “signature”, this paradoxical prediction of quantum theory would seem to have no testable consequences. State vector collapse in that case becomes indistinguishable from a stochastic ensemble description.

The experiment described here provides a means for testing Schrödinger’s paradox. A Michelson interferometer is used to test for the presence of state superposition of a pair of shutters that are placed along the two optical arms of the interferometer and driven by a beta decay source so that either the first shutter is open and the second closed or vice versa. The shutters take on the role of the cat in the Schrödinger paradox.

The experiment that we discuss here has been carried out at SRI International. Under the conditions of the experiment, the results remove the possibility of the existence of macroscopic superposition prior to observation.

1. Introduction

The Schrödinger paradox is among the oldest of the puzzles surrounding the interpretation of quantum mechanics. Like the Einstein–Podolsky–Rosen (EPR) paradox it has engendered a great deal of speculation about our basic understanding of physical reality. Also like the EPR paradox, Schrödinger proposed his paradox to point out that the statistical interpretation of quantum theory must at some level contain a flaw, since it implies the reality of a linear superposition of states even at the macroscopic level – before observation. Moreover, just as in the case of the EPR paradox, the Schrödinger paradox has long been thought of as an untestable consequence of quantum theory, since it relates to the state of a macrosystem just before observation, a state that we know must approach asymptotically to that given by classical mechanics. That is to say, we know that the usual interference effects by which we distinguish the presence of state superposition in atomic processes can be shown to be too small to observe in the case of macroscopic systems. But the development of Bell’s theorem showed us how to test the paradoxical implications of quantum mechanics that had been pointed out by Einstein, Podolsky and Rosen in 1935. Within the limits of our experimental setup, we have now done the same for the Schrödinger paradox.

There are important reasons for doing this experiment. All of us are quite aware of the fact that the existence of a linear superposition of states at the macroscopic level is quite counter-intuitive. Nevertheless, no experiment has ever been done that has yielded results contrary to the literal application of quantum theory. The absence of superposition at the macrolevel prior to observation has not been experimentally demonstrated – and in fact it has generally been thought that such a test was not feasible. This has led to the development of various interpretations of quantum mechanics having to do with the macroscopic reality of quantum states.

A second reason for carrying out an experiment of the present type is that it represents an efficient way to search for the nature of and
cause of state vector collapse. We all know that the machinery that effects state vector collapse, whatever that phrase actually means, must be somewhere between the thing observed and the observer. Indeed, this observer–observed dichotomy has become a rather commonly used phrase in discussions of the measurement problem. But the gap between these two covers a lot of territory. Moreover, we also know that the perponderant opinion is that the transition from the pure state to the mixed state probably takes place at a level above that of the largest coherence that exists for the system being observed. But to focus all our attention at that level at this stage of the game when we still know so little about what causes state vector collapse may not be an efficient way to explore the physics involved. It may be a more efficient strategy to carry out experiments looking for the existence of state superposition at various levels between the atomic level and that of the macroscopic world. Most experiments in this field are designed to examine a cut in von Neumann’s chain between the observer and the observed just above the level of the basic atomic interaction itself. Our experiment goes to the opposite extreme to look for state superposition immediately prior to observation at the macroscopic level.

By doing this we are able to deal experimentally with what has come to be a quite widespread and popular conception of what quantum mechanics has to say about physical reality. The Schrödinger paradox has been used to imply an actual “observer–observed” dichotomy exists as a fundamental aspect of physical reality, and to imply that the observer creates his own reality in the act of observation. It has been used to raise such questions as the “Wigner’s friend” paradox and even to promote speculation that by our observation we may be creating the Big Bang of the universe. If our experiment does nothing more than lay such speculation to rest it will have been more than worthwhile.

At the other extreme, however, we should recognize the possibility that it is through this doorway that some phenomena, heretofore not dealt with by science, may be approached. The existence of consciousness as a phenomenology that lies beyond what we as physicists mean by distance, mass, electric charge and the other constructs of our physical equations cannot be denied. Consciousness surely arises out of something that goes on in the brain of each of us, but yet lies beyond its usual description as a physical object no matter how complex. It may be that if quantum mechanics does require the observer as an essential and irreducible aspect of physical reality, then we may find its proper scientific description to be bound up with an understanding of how state vector collapse comes about. Whatever the likelihood that we will find evidence for this in the experiment we discuss here, it would seem to be worth the effort to look.

The Schrödinger paradox arises because the prescription for writing the general state vector for any system requires that one can sum all possible component states for an unmeasured or unobserved system irrespective of the scale of the system to be observed. As a consequence according to Schrödinger, a cat placed in a box rigged to release a tranquilizer (out of difference to the SPCA – and this writer) if a beta decay occurs in a specified interval of time, or not if the beta decay does not occur, must be represented by a state vector that is the sum of both possible outcomes before a measurement is made on the system. Using Dirac’s notation this would give for the combined state

$$|\Psi\rangle = |\psi_A\rangle + |\psi_S\rangle,$$

where the subscripts A and S refer to the awake and sleep states respectively. Although this is generally regarded to be a preposterous conclusion, there exist no experiments that violate this or any of the basic premises of quantum mechanics. A definitive experiment that would demonstrate that such a superposition of states does not exist would be a significant if not a surprising achievement. On the other hand, since there exist no examples of a violation of the principles of quantum mechanics, it must be considered a viable possibility that quantum mechanics is valid here as well.

It is usually thought that one of two possible
influences causes state vector collapse. One of these is that something happens during the transition from the microscopic realm of the system to the macroscopic realm. Efforts so far to formulate such a suggestion have been unsuccessful—in fact all such proposals that have been reduced to a mathematical prescription have proved to be wrong because they have predicted results at variances with experiments already conducted. Of course we know that on measurement state vector collapse will occur— or will have occurred. If we open the box, we will see the cat either awake or asleep. Some few scientists have suggested that the act of conscious observation causes state vector collapse. Wigner has pointed out that such is a peculiar implication of quantum mechanics, but he has made no effort to formulate what this would mean, if indeed he takes this possibility seriously. Wheeler has pointed out that Bohr specifically "rejected the term 'consciousness' in describing the elemental act of observation... he emphasized that no measurement is a measurement until it is 'brought to a close by an irreversible act of amplification and
until the result is 'communicable in plain language.'" These ideas have not been reduced to mathematical formulation, have not been derived from the Schrödinger equation (note that the Ehrenfest theorem does not cover the case considered here; it does not show that any reduction in the number of states exists in the transition to the microscopic, only that certain kinds of macroscopic processes approach the classical in the limit), and the latter prescription is clearly anthropomorphic—nothing more.

We have carried out an experiment that not only tests the Schrödinger paradox, but has the potential through modest modifications to range the entire gamut of possibilities in order to establish exactly where and how state vector collapse takes place. The experiment makes use of Michelson interferometer (a Mach–Zehnder interferometer could equally be used) in which two shutters, acousto-optical (AO) cells, are placed one in each arm of the interferometer. These AO cells are driven by a quantum mechanical process, specifically, a cesium 137 gamma source driving a bistable multivibrator (flip-flop) circuit in such a way as to gate one or the other of the two possible paths in the interferometer. Thus, knowing the state of the quantum process driving the AO cells we would know that either path 1 was open while path 2 was closed or vice versa. Since we do not know the quantum state driving the AO cells, however, the system must be in both states—that is, in the linear superposition of states. Fig. 1 shows the layout of the experiment.

Although we have replaced Schrödinger's cat with the more manageable AO cells, it is easy to see—as in figs. 2 and 3—that this is a realization of the Schrödinger paradox in which we have

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Fig. 2. Schematic showing that the experiment of fig. 1 is in fact a variant of the Schrödinger paradox arrangement. Here the cat in Schrödinger's thought experiment is shown awake and sitting in the way of arm 1 of the interferometer.
added an interferometer to test the existence of a superposition of pure states or simply the presence of one or the other unknown state of a mixture of states. Now let us look at the equations appropriate to the problem.

2. The cat and the correlation of states

Let us now look at why we should not ordinarily expect to observe any effect with large objects in the first place. The system described by $|\Psi\rangle = |\psi_1\rangle + |\psi_2\rangle$ for which an observation operator $A(x)$ in configuration space would yield $A|\psi_1\rangle = \beta_1|\psi_1\rangle$ and $A|\psi_2\rangle = \beta_2|\psi_2\rangle$ yields for an observation probability $p$,

$$p = \langle \Psi | A^* A | \Psi \rangle$$

$$= (\beta_1^* \langle \psi_1 | + \beta_2^* \langle \psi_2 |)(\beta_1 | \psi_1 \rangle + \beta_2 | \psi_2 \rangle)$$

$$= |\beta_1|^2 + |\beta_2|^2 + \beta_1^* \beta_2 \langle \psi_1 | \psi_2 \rangle$$

$$+ \beta_2^* \beta_1 \langle \psi_2 | \psi_1 \rangle.$$  \hfill (2)

Because our object is large however, the phase factors entering into $\psi_1$ and $\psi_2$ will vary rapidly, so rapidly that the terms $\langle \psi_1 | \psi_2 \rangle$ and $\langle \psi_2 | \psi_1 \rangle$ are for macroscopic objects zero. As a consequence, Eq. (2) reduces to $p = |\beta_1|^2 + |\beta_2|^2$ which is indistinguishable from the classical probabilities for the system.
3. Michelson interferometer

Assume the arrangement shown in fig. 1, but in which both AO cells are always on. This gives us then the standard Michelson interferometer with a CW laser source. Using the subscripts 1 and 2 for the two arms of the device, we write for the state of a single photon $|\Psi_0\rangle = |\phi_1\rangle + |\phi_2\rangle$. For a configuration space photon absorption operator $A(x)$ satisfying $A(x)|\phi_1\rangle = \beta_1|\phi_1\rangle$ and $A(x)|\phi_2\rangle = \beta_2|\phi_2\rangle$ obviously we will have for the probability $p_0$, $p_0 = \langle \Psi_0|A^\dagger A|\Psi_0\rangle$, so that

$$p_0 = |\beta_1|^2 + |\beta_2|^2 + \beta_1^* \beta_2 \langle \phi_1 | \phi_2 \rangle$$

which is formally what we found in eq. (2). However, for photons in an interferometer, terms like $\langle \phi_1 | \phi_2 \rangle$ contribute significantly. We will return to this later. The point here, however, is that it is the presence of these cross terms that lead to the interference effects we observe with an interferometer.

Since we are using a constant wave (CW) laser for our source, the photon is represented by

$$\phi_1 = a_1 e^{i(kx_1 - \omega t_1)},$$

where $x_1$ is the path length for arm number one in the interferometer, $t_1$ is the time of the measurement, $k$ and $\omega$ are the wave number and angular frequency and $a_1$ is a normalization factor. If $\langle \phi_1 | \phi_2 \rangle$ and $\langle \phi_2 | \phi_1 \rangle$ are averaged over complete cycles of $x_1$, $x_2$, $t_1$ and $t_2$, these terms will vanish. With equivalent paths for the two arms of the interferometer, $|\beta_1|^2 = |\beta_2|^2 = \beta^2$ so that we can simply define $p_0 = 2\beta^2$.

4. Michelson interferometer test of the Schrödinger paradox

Now let us look at the the complete problem as shown in fig. 1 in which the state of the quantum mechanical system depends on the coupled gamma decay–photon system. The gates, $G$, are functions of a parameter $B$ of the gamma decay and of the arm of the interferometer in which the gate is located, while the photon representation as before depends on the arm of the interferometer, position and time $t$. We have in general $|\Psi\rangle = |G \rangle \otimes |\phi \rangle$. This gives us

$$|\Psi\rangle = G_1(B, \text{arm } 1)|\phi_1(x_1, t)\rangle + G_2(B, \text{arm } 2)|\phi_2(x_2, t)\rangle. \quad (5)$$

The parameter $B$ has two states which we designate “on” (or “+”) and “off” (or “−”) for arm number one of the interferometer and for the second arm, “off” (or “−”) and “on” (or “+”) respectively, as determined by the logic of the switching circuit. Eq. (5) becomes

$$|\Psi_1\rangle = G_1^*|\phi_1\rangle + G_2^*|\phi_2\rangle + G_1^-|\phi_1\rangle + G_2^-|\phi_2\rangle. \quad (6)$$

As before we write $A(x)|\phi_1\rangle = \beta_1|\phi_1\rangle$, etc. We therefore have

$$A|\Psi_1\rangle = G_1^* \beta_1|\phi_1\rangle + G_2^* \beta_2|\phi_2\rangle + G_1^- \beta_1|\phi_1\rangle$$

$$+ G_2^- \beta_2|\phi_2\rangle. \quad (7)$$

The detection probability function $p_1$ is then

$$p_1 = \langle \Psi_1|A^\dagger A|\Psi_1\rangle$$

$$= \langle (G_1^* \beta_1 |\phi_1\rangle + G_2^* \beta_2 |\phi_2\rangle + G_1^- \beta_1 |\phi_1\rangle + G_2^- \beta_2 |\phi_2\rangle |G_1^* \beta_1 |\phi_1\rangle + G_2^* \beta_2 |\phi_2\rangle + G_1^- \beta_1 |\phi_1\rangle + G_2^- \beta_2 |\phi_2\rangle \rangle, \quad (8)$$

which gives

$$p_1 = |\beta_1|^2|G_1^*|^2 + \beta_1^* \beta_2 G_1^* G_2^* \langle \phi_1 | \phi_1 \rangle$$

$$+ |\beta_1|^2 G_1^* G_1^* + \beta_1^* \beta_2 G_1^* G_2^* \langle \phi_1 | \phi_2 \rangle$$

$$+ \beta_1^* \beta_2 G_1^* G_2^* \langle \phi_1 | \phi_1 \rangle + |\beta_2|^2|G_2^*|^2$$

$$+ |\beta_1|^2 G_2^* G_1^* + \beta_1^* \beta_2 G_2^* G_2^* \langle \phi_1 | \phi_1 \rangle$$

$$+ |\beta_1|^2 G_1^* |\phi_1\rangle + \beta_1^* \beta_2 G_1^* G_2^* \langle \phi_1 | \phi_2 \rangle$$

$$+ \beta_1^* \beta_2 G_1^* G_1^* \langle \phi_1 | \phi_1 \rangle + |\beta_2|^2|G_2^*|^2.$$  (9)
Since the component states such as $\phi_1$ and $\phi_2$ in the linear superposition are the same functions as for the single states alone, we have simply that $G_1^- = G_2^- = G_1^+ = G_2^+ = 0$, and $|G_1^+|^2 = |G_2^+|^2 = 1$. Eq. (9) reduces to

$$p_t = |\beta_1|^2 + |\beta_2|^2 + \beta_1^* \beta_2 \langle \phi_1 | \phi_2 \rangle + \beta_2^* \beta_1 \langle \phi_2 | \phi_1 \rangle,$$

which is the same result as in eq. (3) for the Michelson (or Mach–Zehnder) interferometer result without AO cells.

5. Representation of the photon

Since the laser is not pulsed and since we can assume the switching rate of the AO cells to be much lower than the frequency of the photon, we can write simply

$$A\phi_1 = \beta_1 a_1 e^{i(kx_1 - \omega t)}$$

and

$$A\phi_2 = \beta_2 a_2 e^{i(kx_2 + \omega t)},$$

where we have introduced the factors $a_1$ and $a_2$ as normalization factors for the particular conditions of the experimental arrangement and detection interval. For essentially identical arms in the interferometer, $\beta_1 a_1 = \beta_2 a_2$, so that we can write

$$p_0' = \Psi^* A' A |\Psi\rangle' = \alpha^2 \left[ 2 \Delta x + \int_{x_1}^{x_1 + \Delta x} e^{i k(x_2 - x_1)} dx_1' \right.$$

$$+ \int_{x_2}^{x_2 + \Delta x} e^{i k(x_1 - x_2)} dx_2' \right],$$

where $\Delta x$ is the thickness of the photographic film layer and where we have incorporated the time interval of the measurement in our normalization factor $\alpha$. The primes indicate that the probabilities represent a measurement over a time that is long with respect to the photon frequency. Of course, for thin films we have simply

$$p_0' = \alpha^2 \left[ 2 + e^{i k(x_2 - x_1)} + e^{i k(x_1 - x_2)} \right].$$

Therefore, in the absence of any formalism that would prescribe state vector collapse below the macroscopic level, our calculation predicts the presence of interference fringes in the present experiment despite its counterintuitiveness.

Therefore, a failure to detect a robust interference pattern will show that the experimental results are in disagreement with our theoretical prediction.

The converse outcome holds equally remarkable significance. The occurrence of interference fringes would mean that the linear superposition of state holds before observation even on the macroscopic scale.

6. The apparatus

The apparatus consists of a simple Michelson interferometer with optical switches in the relay arms. A schematic diagram of the arrangement is shown in fig. 1.

The polarized output beam from a 6328 Å CW helium-neon single mode laser is attenuated by a factor of $10^{-5}$ so as to produce a beam of $1.3 \times 10^{-14}$ W, approximately $4.17 \times 10^4$ photons per second intensity. The attenuation is achieved by a combination of the deflected beam intensity reduction, neutral density filters and a polarizer. The light incident on the beam splitting is polarized with the electric vector perpendicular to the plane of fig. 1.

The light is passed to a beam splitter which produces beams of nearly equal intensity. Each arm of the interferometer contains an acousto-optical modulator consisting of a TeO$_2$ crystal coupled to an ultrasonic piezoelectric transducer. When no input voltage is applied to the transducer, light travels through the crystal undeviated. With an 80 MHz rf signal applied to the transducer, the resulting acoustic waves in the
crystal diffract the light beam by twice the Bragg angle, which is 5.9 mrad for the devices used. It is this diffracted beam that is used to obtain interference in the interferometer. Turning the transducer off turns the AO cell off. These acousto-optical devices exhibit a rise and fall time of 120 ns when switching a beam of 0.75 mm diameter. Most of the power (80%) goes into the first-order diffracted beam, which leaves the device at an angle of 11.8 mrad to the zero-order undiffracted beam, while smaller amounts of power are deposited into the higher orders. As stated, it is the first-order diffracted beams that are reflected off the interferometer mirrors and back through the modulators where a second diffraction occurs. Only the first-order beams are shown in fig. 1 for clarity. The zero-order and higher-order diffracted light is absorbed by various beam stops. Thus the acousto-optical modulators act to chop the light in the interferometer arms with a contrast ratio adequate to assure that none of the photons reaching the film will have passed through an “off” shutter.

The switched beams from the interferometer arms are recombinated in the beam splitter and are deposited on a high speed photographic emulsion. The beam divergence of the laser and the geometry of the apparatus are chosen so as to result in a 2 mm diameter image on the film with approximately four linear fringes visible in this area.

The acousto-optical modulators are driven by switched 80 MHz oscillators which are in turn gated on and off by the outputs of a bistable multivibrator, or flip-flop, so that one modulator is on while the other is off. A delay of 150 ns is introduced into the gating signals applied to the rf drivers so that one shutter does not start to open until the other has closed, thus ensuring that at no time are both interferometer arms open. The flip-flop is clocked by pulses from a photomultiplier, which have been suitably amplified, shaped and discriminated. The photomultiplier looks into a sodium iodide scintillator crystal. With a cesium 137 source of approximately 30 μCi placed 2 cm from the scintillator, the pulses which clock the flip-flop have a mean repetition rate of 118 kHz.

With an average photon rate of \(4.17 \times 10^4 \text{s}^{-1}\) emerging from the attenuator, there is an average of less than one photon in each arm of the

Fig. 4. Photograph of the laboratory layout.
interferometer during each period for which the acousto-optic modulator in the arm is open. The resulting low intensity image is recorded on Kodak 2415 technical pan film hypersensitised in forming gas prior to exposure. A high speed Polaroid film has also been used. A photograph of the laboratory setup is shown in fig. 4.

7. Test runs

In order to verify that the apparatus properly discriminates between the two possible experimental outcomes, test runs were made with the logic circuit connected so that both shutters would either be on or off to make certain that the apparatus could give interference patterns. This run gives us a reference interference pattern that we can use to judge the results of our experimental run. The average shutter rate in this case was the same as for the final experimental run. The resulting photograph is shown in Fig. 5.

8. Experimental results and conclusions

Fig. 6 shows the experimental outcome for a photon rate of $4.17 \times 10^4$ s$^{-1}$. The figure speaks for itself. There is no interference pattern present in the figure. The figure clearly shows the absence of the interference fringes predicted by our formalism.

This absence of interference fringes in the experimental run is a result that, although expected on the basis of commonsense, we nevertheless interpret as a prima facie case that quantum theory may be violated. The result is important because it provides a starting point for us in our search for the cause and experimental meaning of state vector collapse. These results are also important because they are related to questions about the Schrödinger paradox.

We do not yet know just how and where state vector collapse occurs - nor do we know what this even means. Our experiment does not solve or remove the measurement problem. If anything, it deepens the problem. We must find out

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Fig. 5. Test run in which both shutters are either open or closed at the same time to assure a conventional interference pattern. Photon rate was $4.17 \times 10^4$ s$^{-1}$, less than one photon in each arm of the apparatus at any moment.
the mechanics of state vector collapse. The present experiment provides a basic plan for future experiments to search out how state vector collapse occurs and to give us a clear understanding of just what state vector collapse entails. It gives us the tool we need to find just where to cut von Neumann’s chain.

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